Multi-criteria games

September 14, 2018

Introduction

- Who does not want to be rational?
- What does rationality mean?

- Philosophy, economics, statistics, computer science, ...
- Ask economists for formal models

Introduction

In economics, rationality = set of axioms
(Rationality in choice = Subjective Expected Utility maximization)

1. The preference relation $\preceq$ is a total preorder, $f \preceq g$
2. $f_E^h \preceq g_E^h$ if and only if $f_E^{h'} \preceq g_E^{h'}$, for all f, g, h, h'
3. Etc.

Criticisms of SEU - Normative part

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- Ellsberg:

	R	B	Y
r	100	0	0
b	0	100	0
ry	100	0	100
by	0	100	100

$R + B + Y = 90; \ R = 30$

Criticisms of SEU - Normative part

"The Bayesian approach is quite successful at representing knowledge, but rather poor when it comes to representing ignorance. When one attempts to say, within the Bayesian language, 'I do not know', the model asks, 'How much do you not know? Do you not know to degree .6 or to degree .7?' One simply doesn't have an utterance that means 'I don't have the foggiest idea'." [Gilboa et al., 2012]

"Justification of beliefs by evidence offers a criterion for rationality that need not rank highly specified beliefs as more rational than less specified ones." [Gilboa, 2015]

Maxmin Expected Utility

- Ellsberg:

	1/3	0	2/3
	1/3	2/3	0
	R	B	Y
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b	0	100	0
ry	100	0	100
by	0	100	100

$R + B + Y = 90; \; R = 30$

$$\operatorname*{argmax}_{a \in A} \min_{P \in \Gamma \subseteq \Delta} E_P[u|a] = r$$

$$\operatorname*{argmax}_{a \in A} \min_{P \in \Gamma \subseteq \Delta} E_P[u|a] = by$$

Decision Criterion: Decision Problems $\rightarrow$ Actions

Classic Evolutionary Game Theory

- A single, fixed (symmetric, simultaneous-move) fitness game
- A population of agents, the players, of some given player types, e.g., *I* and *II*
- Some random or assortative matching

G^0	*I*	*II*
I	1;1	2;5
II	5;2	0;0

$G^0, G^0, G^0, \ldots$

1. Static analysis: ESS, NSS, equilibrium states, etc.
2. Dynamic analysis: RD, RMD, rest points, asymptotic stability, Lyapunov stability, basins of attraction, etc.

Evolution in Richer Environment

- A SET $\mathcal{G}$ of (symmetric, simultaneous-move) fitness games
- A population of agents, the players, of some given player type, e.g., WHAT?
- Some random or assortative matching

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G^2	I	II
I	3;3	0;1
II	1;0	1;1

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$G^1, G^2, G^3, G^4, \ldots$

$\mathcal{G}$:

	I	II
I	a	b
II	c	d

What is a player type?

In general, a function t such that $t(G) \in A_G$, where $G \in \mathcal{G}$ and A_G is the set of available actions in game G.

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Or maxmin:

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II	5;2	0;0

	I	*II*
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Evolution of decision criteria.

Multi-criteria games

1. Ω states
2. Λ parameters
3. I players
4. for each $i \in I$:
   - A_i set of actions
   - T_i set of criteria
   - S_i set of signals
   - $\tau_i : \Omega \to T_i$ criterion-assignment function
   - $\varsigma_i : \Omega \to S_i$ signal function
   - $u_i : A \times \Omega \to \mathbb{R}$ utility function
5. a probability measure P_λ over Ω for each $\lambda \in \Lambda$

Linear Regret

	R	B
r	1000	0
b	0	3000

$|R+B| = 10$; 1 blue, 7 red

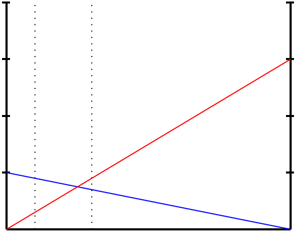

Figure: $r, b \in \text{argmin}_{a \in A} \max_{p \in \Gamma} E_p \left[\max_{a' \in A} u(s,a') - u(s,a)\right]$

Nonlinear Regret

	R	B
r	1000	0
b	0	3000

$|R + B| = 10$; 1 blue, 7 red

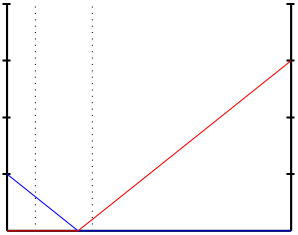

Figure: $b \notin \text{argmin}_{a \in A} \max_{p \in \Gamma} \max_{a' \in A} E_p[u(a', s)] - E_p[u(a, s)]$

Results

$T_i = \{\text{Mm}, \text{NRm}\}$

In 2×2 games, NRm is the only evolutionarily stable type in the population.

Results

$T_i = \{\text{flatEU, Mm, LRm, NRm}\}$

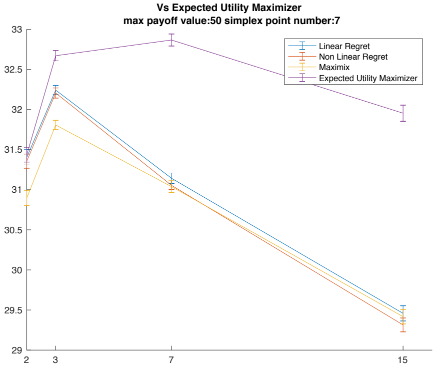

Figure: 10000 games with possible fitness values in the set $\{0, \ldots, 50\}$

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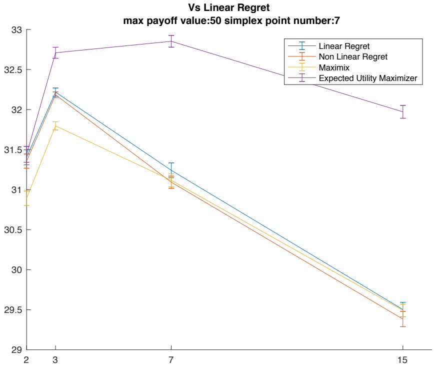

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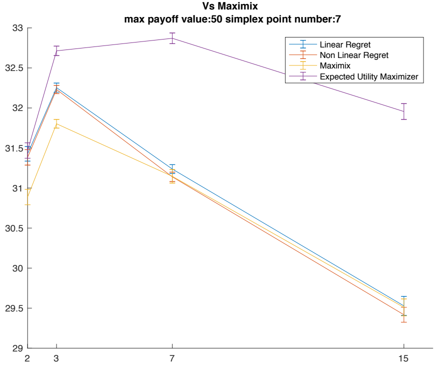

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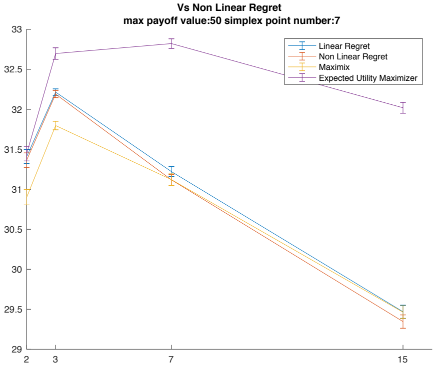

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Biology and behavioral ecology

> By focusing on expressed behavior and neglecting the underlying mechanism, behavioral ecologists unwittingly adopt the behavioral gambit, extending the phenotypic gambit beyond its accepted remit. [...] Natural environments are so complex, dynamic, and unpredictable that natural selection cannot possibly furnish an animal with an appropriate, specific behavior pattern for every conceivable situation it might encounter. Instead, we should expect animals to have evolved a set of psychological mechanisms which enable them to perform well on average across a range of different circumstances. [Fawcett et al., 2013]

Conclusion

To conclude:

- Extensions of EGT to multigames are informative and interesting.
- Much more to be studied: inferring and learning.
- Ecological vs axiomatic assessment of rationality.

Thanks for your attention.